OPTIMIZATION OF HYDRAULIC FRACTURING
OF TIGHT GAS FORMATIONS IN HORIZONTAL WELLS

By

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ABSTRACT

The discoveries of significant resources of oil and gas in the Barnett, Bakken, Marcellus, and other shale formations have presented an opportunity for national petroleum independence and possible export in the relatively near future. These formations are considered to be “unconventional” resources because of the extremely low formation permeability. This means that traditional oil and gas recovery mechanisms are not economical in these formations. With the development of methods for performing multiple hydraulic fractures in horizontally drilled wells in reasonable periods of time economic production of these formations has been seen throughout the world. However, little is known about the optimal spacing of these fractures in horizontal wellbores – how close together should they be to optimize the net present value (balancing costs of drilling, stimulation and production with future revenue). This paper outlines current hydraulic fracturing propagation theory and describes fracture interaction phenomena such as fractures linking-up or one set of fractures inhibiting the growth of others. This leads to insights related to hydraulic fracture spacing. The optimization of hydraulic fracturing spacing was simulated using mShale™ from Baker Hughes. It was discovered however that the interference of multiple fractures in a defined zone can be modeled with the mShale software package and it serves as a good pseudo design estimation of optimum fracture spacing.
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1. INTRODUCTION

There have been many research studies to describe hydraulic fracturing techniques and the theory behind this widely used practice. Hydraulic fracturing, or “fracking”, as it is commonly referred to, is the process of fracturing tight (low permeability) hydrocarbon-bearing rock in order to increase the large scale permeability and connectivity to a wellbore. With the widespread use of directional drilling coupled with hydraulic fracturing in multiple, discrete sections of a wellbore some unconventional oil/gas supplies have been determined to be economically recoverable. Representative prolific shale formations include the Bakken formation, (Canada, Montana, and North Dakota,) the Barnett formation, (Texas,) the Marcellus formation, (Ohio, New York, and Pennsylvania.), to name a few. The purpose of this study is evaluate optimizing the hydraulic fracture spacing in horizontal wellbores - to increase productivity and to minimize hydraulic fracturing capital costs.

1.1. General Characteristics of Shale Formations

For many years the petroleum industry has considered shales as “seal rock formations” or a formation that has trapped oil and/or gas in an acceptable reservoir. This assumption was traditionally made because shales have low permeability, and low porosity that inhibit the flow of fluids through the formation (vertically or laterally). Shales are a sedimentary rock that is indurated by burial and compaction. Typical shales are fine-grained and consist of clays, muds, and as well as detrital calcite and quartz. Prior to 1973 the petroleum industry had discovered hydrocarbons in shale formations such as the Marcellus formation. However, there was little success in recovering economical volumes of natural gas at sustainable rates. This view changed when successful hydraulic fracturing in the Barnett shale formation started to suggest acceptable results. The first Barnett shale well was drilled in the early 1980s.
Economic production came in the 1990s and 2000s. Since then a number of different shale formations rich in hydrocarbons have been drilled leading to the recent energy boom in the United States.

1.2. **Economic Production of Shale Formations by Advanced Drilling Technology**

Hydrocarbon production from shale formations has been made possible through breakthroughs in drilling, completion and simulation technology. Some of the main technologies affording economical production in shale formations are: 1). horizontal drilling, 2). economic yet aggressive hydraulic fracturing, and 3. isolation technologies. A brief discussion of each of these technologies follows.

1.2.1. **Horizontal Drilling**

Production from a wellbore is directly related to the contact area of the wellbore to the rock that contains producible hydrocarbons. In traditional vertical wells the contact area with an unfractured wellbore is minimal since it is only able to intersect the total height of the target formation. For gas-rich, low permeability formations hydrocarbon production was usually not economical unless a major natural fracture system was serendipitously intersected. With the use of improved steering technology at the bit, horizontal drilling advanced and made it possible to interact with much greater reservoir volumes.

The advantage of horizontal drilling is that the drilling operator can deviate from a vertical trajectory and intersect the vertically thin but laterally extensive gas-bearing reservoirs. Horizontal drilling substantially increases the contact area with the wellbore as compared to a vertical well. According to the U.S. Energy Information Administration “Horizontal drilling lets producers access far more natural gas from
relatively thin shale deposits. Directional drilling alone is not enough. Evolution of hydraulic fracturing protocols are a second piece of the picture since very few unstimulated (not hydraulically fractured) horizontal wells will economically produce.

1.2.2. Hydraulic Fracturing

Hydraulic fracturing has contributed to economic hydrocarbon production from low-permeability reservoirs. In a classic analysis of hydraulic fracturing “the rock was regarded as an elastic medium, and by assuming that the well was under a plane-strain condition, the breakdown pressure for the wellhole was determined by equating the circumferential stress at the well surface to the tensile failure stress of the rock medium.” The breakdown pressure is defined as “the pressure at which the rock matrix of an exposed formation fractures and allows fluid to be injected. The breakdown pressure is established before determining reservoir parameters.” Despite the importance of hydraulic fracturing, the most important technological advances have often be related to methods for isolating individual sections of a wellbore, carrying out a fracturing stage, moving back up the hole, isolating another section and creating another fracture.

1.2.3. Isolation Technologies

Before recent advances in isolation technologies hydraulically fracturing multiple zones in new wellbores was time intensive and consequently expensive. With newer isolation technologies service companies fracture horizontal wellbore in numerous discrete stages rather than all at once. By placing an inflatable, mechanically expandable or chemically swellable rubber elements (packers) in specific locations in a wellbore it is relatively routine to restrict fluid flow to that isolated area of the
wellbore and build up surface-generated hydraulic pressure. After hydraulic fracturing in one zone is complete it is isolated from the next upstream target area and this, subsequent zone is hydraulically fractured; and so on. Without isolation technologies such as plug and perf and the use of packers hydraulic fracturing could not be done economically.

1.3. Opportunities for Hydrocarbon Recovery from Shale Formations

Over the past couple of decades most of the traditional, oil/gas reservoirs have been located, drilled, and produced. These were relatively shallow, with reasonable permeability and porosity. Along with depletion of these traditional reserves have come theories that the world would run out of hydrocarbons in the near future. This was projected by the American geophysicist M. King Hubbert in his famous theory; known as Hubbert’s Peak or peak oil. He argued that worldwide hydrocarbon production would peak and then decline to 0 as a bell-shaped curve - starting as early as 2020 in the United States. What the Hubbert’s Peak theory failed to take into consideration were the technological advances in drilling and oil/gas enhancement techniques making historically defined “unconventional reservoirs” available for economic production in present day. In fact it is projected that the United States will become the world leader in hydrocarbon production by 2020 and will become energy independent by 2030 largely due to the exploitation of low permeability, hydrocarbon-rich reservoirs. What are these reservoirs like? Three representative reservoirs are described; the oil-producing Bakken formation, the Barnett shale, and the Marcellus shale.
1.3.1. **Bakken Shale Formation**

The Bakken shale formation is located in Montana, North Dakota, and Southern Canada. It was discovered by J.W. Nordquist in 1953 and is currently one of the largest onshore oil recovery operations in the world. Extensive drilling and production of the Bakken formation - in the most recent era - started in 2008 with the arrival of modern hydraulic fracturing techniques. As a matter of perspective, since late 2010, oil production from the Bakken Shale has been so significant that it has overwhelmed the shipping infrastructure. This has forced companies to use different methods of moving the crude to refineries around the United States - such as by rail or new pipelines. The Bakken formation is estimated to have a “total Original Oil in Place, OOIP, of 169,113,151,604 bbl of which 3,962,120,299 bbl have been identified as recoverablev”. All oil currently being produced in the Bakken formation is classified as light crude. Surprisingly, such of the production comes from non-shaley members of the Bakken; albeit still low permeability, characteristically this is a low permeability, low porosity producer with a high Young’s modulus (indicating the resistance that the rock provides to any sort of deformation, including the pressure in a hydraulic fracture. One source indicates that Young’s Modulus of the Bakken shale is $7.69 \times 10^6$ psi$^vi$” with an average permeability of “$0.04 \pm 0.005 \ mD$ and an average porosity of $5\% \pm 0.1\%vii.” The Bakken formation will be used as a type shale oil reservoirs containing liquids.

1.3.2. **Barnett Shale Formation**

The Barnett shale formation, located in Texas, was one of the first successful unconventional reservoirs made available to economic production with the use of hydraulic fracturing in 2008. The Barnett shale is mainly a natural gas producer with
an estimated Original Gas In Place, OGIP, of "30 tcf." Generically, the properties are similar to the Bakken formation. One source reports an average Young’s Modulus of “$6.25e + 06 \, psi$” with an average permeability of “0.08 mD and an average porosity of 6%.” In some instances, gas production is accompanied by condensates. Condensates are classified as a low-density mixture of hydrocarbon liquids that are sometimes recovered during the production of a wet natural gas field. As the raw natural gas rises from the reservoir to the surface these condensates can turn into liquid if temperatures in the wellbore drop below that of their respective dew points. Condensates add to the economics of a natural gas well as they can be sold as a high price, helping the operating company recover its investment and operating costs quicker.

1.3.3. **Marcellus Shale Formation**

Finally, the Marcellus shale formation - located in Ohio, New York, and Pennsylvania - is one of the largest United States shale formations that have been exploited to date. The Marcellus shale is considered to mainly contain natural gas with an estimated OGIP of 141 tcf. Like its counterparts, described above permeability and porosity are low and Young’s Modulus is somewhat lower. One source indicates an average Young’s Modulus of the Marcellus shale is “3.49 x 10^6 psi” with an average permeability of 0.001 mD” and an average porosity of “7%.”

1.4. **Sustainability of Hydraulic Fracturing**

When implementing new technologies that have a potential impact on the environment and the quality of life of stakeholders living nearby it is important to appreciate if the technologies are sustainable. Over recent years hydraulic fracturing has
received a lot of attention from the mainstream media along with some complaints from local citizens that it pollutes potable water sources, accelerates PM2.5 and similar. In order to understand if hydraulic fracturing is a sustainable process it will be evaluated from an economic, an environmental, and a long-term stability standpoint.

1.4.1. Economic Sustainability of Hydraulic Fracturing

Hydraulic fracturing is currently necessary in order for the world to be able to continue to produce hydrocarbons to be used as energy and chemical feedstocks. Over the past century, a majority of the dominant conventional reservoirs have been located and produced to near depletion - diminishing the volume of readily available hydrocarbons throughout the world. In order to meet the increasing global demand for oil and natural gas it is important that new, modified and refined stimulation technologies be embraced cursory economic analysis one can assert that hydraulic fracturing can be economically sustainable as only approximately “$50,000 to $1,000,000” of added capital expenditure is needed to fracture a new or existing well; these expenditures can easily be recovered when the well is put into production. It should be noted that by increasing the capital expenditure per unconventional wellbore could and most likely would increase the price of a barrel of oil or the price of natural gas per dekatherm as operating companies will probably pass the extra costs to the consumer.

1.4.2. Hydraulic Fracturing is Environmentally Benign

As hydraulic fracturing receives more and more attention from media it is vital that oil and gas companies, governmental officials, researchers, and citizens understand if hydraulic fracturing can be performed in a way that is environmentally benign. In
order to better understand the impacts of hydraulic fracturing the United States’ Congress requested the EPA to carry out an independent scientific study be performed on hydraulic fracturing and its potential impact on drinking water resources. The charter of this research project the full life cycle of water used in hydraulic fracturing, ensuring that proper techniques are implemented to ensure the integrity of the nation’s water supply.

The EPA’s hydraulic fracturing study began in November 2011 and consists of understanding the hydraulic fracturing water cycle as described in Figure 1:

Figure 1: The Hydraulic Fracturing Water Cycle

Within the hydraulic fracturing water cycle, the EPA will determine if hydraulic fracturing can be done in such a way that does not affect local aquifers and wastewater treatment facilities. One of the key points to this hydraulic fracturing evaluation will be to review well injection techniques and ensure that all wells are cased in areas where naturally occurring aquifers are found, isolating them from the potential of being contaminated with natural gas or liquid hydrocarbons as illustrated in Figure 1 point 3. Another area of focus is ensuring that hydraulic fractures are not allowed to propagate
upwards through overlying formation fractures linking the reservoir to an active aquifer.

In December 2012 the EPA released a progress report on the “Study of the Potential Impacts of Hydraulic Fracturing on Drinking Water Resources” disclosing some of their progress and the number of different geographic locations involved in the study. The EPA study covers 333 wells across the United States as well as areas where water containments have been reported by local agencies. At this time the EPA has released this statement in their progress report, “Information presented as part of this report cannot be used to draw conclusions about potential impacts to drinking water resources from hydraulic fracturing.” However, according to the progress report, the EPA has not found any evidence suggesting that hydraulic fracturing is immediately harmful to the citizens of the United States at this time and has, for the time being, not suspended hydraulic fracturing operations nationwide. It therefore can be assumed that hydraulic fracturing is environmentally benign and a final decision about its sustainability will be made when the EPA files their final results in the first quarter of 2014.

1.4.3. **Long-Term Stability of Natural Gas Supply**

In order for hydraulic fracturing to be considered a sustainable process it is important to project if it will lead to long-term stability of natural gas production to meet ever increasing worldwide energy demand. By utilizing hydraulic fracturing technology there is new availability of unconventional hydrocarbon resources. This should help to keep energy prices down while improving the strength of the worldwide economy. After analyzing the economics and environmental impacts of hydraulic
fracturing it seems to be a sustainable and necessary industrial process in order to ensure that the United States can become energy independent by the year 2030 while providing necessary resources to developing countries to improve the quality of life of their respective citizens.

1.5. Upcoming Challenges to Hydraulic Fracturing

There are three main challenges that will need to be addressed by geologists and engineers in order to improve the hydraulic fracturing effectiveness:

1. Create proper stage size models for different rock formations
   i. Stage size is defined as how many different number of perforations that are needed to effectively fracture a given length of a known formation without adding excess costs. By creating stage size models for different rock formations the hydraulic fracturing process may be optimized and unnecessary perforation and fracking costs can be reduced improving the overall economy of the well.

2. Optimize cluster geometries for rock perforating
   i. Before fracking occurs it is vital to understand the direction of the in situ stresses such that proper shape charges and cluster geometries can be selected to minimize costs.

3. Optimize hydraulic fracturing spacing in different rock formations
   i. Hydraulic fracturing spacing is defined as the given length between the start of one fracture to the next. If the fracture spacing decreases the effects of fracture interactions will increase causing a potential interference is adjacent fractures.

By tackling these challenges the capital expenditures related to hydraulic fracturing can and will be reduced while at the same time increasing the integrity of the wellbore(s). The
focus of this research paper is to address the optimization of hydraulic fracturing spacing in shale rock formations.

1.6. Thesis Overview

The following chapters address the theory of hydraulic fracture optimization and describe the simulation methods used in the spacing evaluations that were carried out. Chapter 2 summarizes the theory of hydraulic fracturing and indicates why proper spacing is vital for achieving maximum fracture surface area in contact with the wellbore. Chapter 3 introduces the simulation model and the setup of the commercial simulator data that were used for Meyer and Associates’ mShale software. Chapter 4 is a comprehensive discussion of the simulation results. Chapter 5 provides conclusions and recommendations for future work.

2. Proposed Theory of Hydraulic Fracture Interactions

When a geological formation, shale or otherwise, is subjected to pressures high enough to cause hydraulically fracturing, it is classically assumed that a fractures ability to propagate is independent of other fractures adjacent hydraulic fractures being concurrently pressurized. However, it has been observed that there are cases in which fractures do not propagate as far as designed. This implies that there is one or more variable within the hydraulic fracturing “system” that is inhibiting certain parallel, concurrently-propagating fractures from propagating equally. When these fractures do not perform as planned the operating company loses money and well productivity that in turn negatively affects the well’s economic returns. It is hypothesized that certain fractures begin propagating must faster than adjacent fractures. When this occurs the pressure in the propagating fractures affects the local in-situ stresses acting to close the slower propagating fracture. Alternatively, it has been argued that this is similar to increasing the effective Young’s Modulus of the system and therefore “inhibiting” proper fracture propagation. By
increasing the fracture spacing in the shale formation this fracture “inhibition” can be mitigated, but the overall economics are impacted if too few fractures develop – production is nominally proportional to the fracture surface area that is hydraulically generated.

2.1. Definition of Research Parameters and Key Assumptions

The optimization of hydraulic fracture spacing in shale formations is a complex problem that will require years of research before it is solved. The scope of this research project was to identify key research parameters and to simplify the optimization of hydraulic fracturing spacing by using mShale™ coupled with a set of reasonable assumptions in order to provide a good first pass solution. The basis of this approach relies on the following assumptions:

1. Fractures do not propagate in any geologic formation other than the shale zone in which they are completed.
2. Simulations considered concurrent parallel growth of between one and fifteen planar, vertical fractures.
3. All fractures are assumed to propagate following a hyperbolic geometric growth pattern.
4. The fracture plane is two dimensional (constant height). The height is the vertical extent of the fracture – in this case, this is taken to be the true vertical thickness of the shale being fractured. Height remains constant but the length and width of each fracture vary with the amount of fluid that is injected.

The key parameters used in this study were the total height of the desired fracture, $H \ (ft)$, the total fracture spacing length, $\delta \ (ft)$, the minimum horizontal in-situ stress, $\sigma_{H\text{Min}} \ (psi)$, and the Young’s Modulus of the formation, $E \ (psi)$. By using these parameters it is hypothesized
that a dimensionless number and dimensionless correlation can be discovered that would allow a quick analytical approach to optimizing fracture spacing within a shale formation.

2.2. Theoretical Qualitative Outcome

In the general wellbore system hydraulic fracturing fluid is introduced into a horizontal wellbore at high pressure and then enters into each of the hydraulic fractures defined by the user. As the pressure increases in each of these fractures, it is assumed that they will each grow orthogonally to the wellbore. As the length of each fracture increases it is probably that they will reach a point in which the fractures will begin to “inhibit” the growth of interior fractures. This concept is illustrated Figure 2 below where the blue arrows represent the first and third fracture and the red arrow represents the inhibited second fracture.

![Figure 2: Inhibition of an interior fracture during a generic hydraulic fracturing treatment with three concurrently pressurized fractures.](image)

This “inhibition” is expected to begin when the fracture half-lengths, $x_f \text{ ft}$ become greater than the initial fracture spacing, $\delta \text{ ft}$. It is hypothesized that there will be only second order fracture interaction or “inhibition” as long as $x_f < \delta$. 
In Figure 2, as the first and third fractures continue to grow, their pressures and fracture interactions will increase to a point that the second fracture will no longer be able to propagate - thus creating an ineffective fracture. This inefficiency could have been avoided if the initial perforations were set far enough apart. The two-dimensional planar proxy that was adopted for a hydraulic fracture presumes an increase in pressure with an increase in injected volume (see for example, Perkins and Kern, 1963??) This pressure must overcome the minimum horizontal in-situ stress that is trying to close it and overcome frictional resistance associated with fluid flow along the fracture length. As the pressures increase the effective Young’s Modulus of the system around the second fracture would increase, most likely proportionally, along with its effective stiffness thus “inhibiting” its ability to grow.

3. Experimental Setup of Simulation using mShale

In order to solve this complex problem, it a simulation package was selected to test the optimized hydraulic fracture spacing hypothesis provided above. After reviewing a number of different hydraulic fracturing simulation packages available to the University of Utah it was decided that the mShale program developed by Baker Hughes would be best suited for this type of simulation. The information found in Appendix E describes how the hydraulic fracturing model was set up in mShale and justifications to why these settings were selected are provided.

3.1. Variables for Hydraulic Fracturing Simulation

For this hydraulic fracturing simulation the two parameters, variables, were:

1. Number of Major Vertical Fractures
2. Young’s Modulus of the shale

The purpose of varying these parameters was to derive an empirical correlation or dimensionless number that could be useful for optimizing the spacing of hydraulic fractures
(cluster or stage spacing – but you need to explain these terms). Table 2 summarizes the simulation matrix:

Table 1: Simulation matrix of all runs for spacing optimization

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Number of Vertical Fractures</th>
<th>Young’s Modulus of Shale (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2 x 10^6</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2 x 10^6</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2 x 10^6</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2 x 10^6</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>2 x 10^6</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>2 x 10^6</td>
</tr>
<tr>
<td>7</td>
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<td>4 x 10^6</td>
</tr>
<tr>
<td>18</td>
<td>15</td>
<td>4 x 10^6</td>
</tr>
</tbody>
</table>

3.1.1. Desired Outputs from the Hydraulic Fracturing Simulation

From each of the eighteen different simulations shown in Table 2, the three key output parameters were:

1. Height of the vertical fracture(s) (ft)
2. Half-length of each fracture(s) (ft)
3. Net pressure of the fracture(s) (psi)
*Net pressure is defined as the $P_{\text{net}} = P_{\text{fracture}} - P_{\text{closure}}$

4. Results and Discussion

The simulations modeled fracture interactions that occurred in a single cluster in a one stage system and how those interactions are affected by changes in Young’s Modulus or the number of major vertical fractures specified. From these data, empirical relationships were established and a new empirical correlation was discovered and quantified by the creation of a new dimensionless number, the Modified Young’s Modulus $mYM$.

4.1. Optimization of Hydraulic Fracturing Spacing

Initially, it was intended to develop an empirical correlation that could be used to optimize the cluster spacing in a hydraulic fracturing system. Calculations indicated that all fracture dimensions were equal. After investigating this, it was discovered that the code used solves all hydraulic fractures independently, thus ignoring propagation and interactions of adjacent fractures entered manually. This hypothesis was validated by running multiple simulations at different spacing lengths, both in the Zones window and the Characteristics tab in the Zone Data window; resulting in identical output for each of the six different cases. MShale was used to optimize a single hydraulic fracturing system by applying and modifying other variables and conditions.

4.2. Young’s Modulus and Hydraulic Fracturing Propagation

As is well known in the petroleum literature, Young’s Modulus of the targeted formation (shale in this instance) directly affects the fracture’s or fractures’ vertical propagation, width, the net pressure of the system, as well as the fracture efficiency. Young’s Modulus, or elastic modulus, is defined as “the ratio of the stress in a body to the corresponding strain.” Geological formations with a high Young’s Modulus are “stiffer”
and resist deformation (development of fracture width) more so than lower modulus materials. By modifying the Young’s Modulus of the shale formation being fractured from $2 \times 10^6$ to $4 \times 10^6$ psi the following three empirical observations were made (in accordance with analytical relationships from Perkins and Kern, 1963, and numerous others):

1. Fracture height increases as a function of increasing Young’s Modulus.
2. The net pressure of the system increases as a function of Young’s Modulus. The net pressure is the pressure at the fracture mouth minus the far-field principal stress acting to close the fracture.
3. The fracture efficiency decreases as a function of the Young’s Modulus.

*Efficiency is the volume of the fracture divided by the total volume of fluid pumped – the difference between the two values being the volume of fluid that is lost to the formation by leakoff – flow into the porous medium intersected by the fracture surfaces.

Although the Young’s Modulus of the certain formation listed in chapter 1 were not tested within this simulation it is believed that their values can be reasonably extrapolated from the collected data set located in Appendix B.

4.2.1. **Relationship between Young’s Modulus and Fracture Height**

After analyzing and plotting the data, it was determined that the height of the fracture(s) was dependent on Young’s Modulus. While in-situ stress and its vertical variations is the dominant control on vertical height growth, there was no vertical variation of the minimum horizontal stress in these simulations. It is possible to envision increased height growth with a higher modulus. A higher
modulus material resists width development. With conservation mass, this means other dimensions will need to increase commensurately. This phenomenon is exaggerated when there are nearby pressurized fractures, as seen in Table 3. Table 3 is one way to compare vertical growth as it is influenced by two relatively high modulus values and by multiple, parallel, pressurized fractures. By comparing the value of the fracture height for a formation with a modulus of $4 \times 10^6$ psi and subtracting that from the value of the fracture height for a formation with a modulus of $2 \times 10^6$ psi and then normalizing it with the value of the fracture height for a formation with a modulus of $2 \times 10^6$ psi the values in Table 3.

Table 2: Relative Fracture Height Growth for Doubling the Modulus in the Presence of Adjacent Pressurized Fractures

<table>
<thead>
<tr>
<th>Number of Fractures</th>
<th>Fracture Height Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2%</td>
</tr>
<tr>
<td>3</td>
<td>2%</td>
</tr>
<tr>
<td>6</td>
<td>3%</td>
</tr>
<tr>
<td>9</td>
<td>3%</td>
</tr>
<tr>
<td>12</td>
<td>3%</td>
</tr>
<tr>
<td>15</td>
<td>4%</td>
</tr>
</tbody>
</table>

Table 3 shows that there is a relationship between an increase in Young’s Modulus and the fracture height, however; it is very small. Table A1 in Appendix A shows non-normalized values for height and length growth. Specific relationships are complicated by fluid loss. The fracture height growth also depended on the number of major fractures that were pressurized. With more fractures, the overall dimensions were smaller but the relative
change in height increased because the effective modulus of the overall system increased (refer to Yew et al., 1993).

4.2.2. Relationship between Young’s Modulus and Net Pressure

There is another relationship between Young’s Modulus and the net pressure of the system. Table 4 was developed by normalizing in the same manner as used for height growth.

Table 3: Net Pressure Increase with the Number of Vertical Fractures

<table>
<thead>
<tr>
<th>Number of Vertical Fractures</th>
<th>Net Pressure Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>63%</td>
</tr>
<tr>
<td>3</td>
<td>66%</td>
</tr>
<tr>
<td>6</td>
<td>65%</td>
</tr>
<tr>
<td>9</td>
<td>64%</td>
</tr>
<tr>
<td>12</td>
<td>64%</td>
</tr>
<tr>
<td>15</td>
<td>64%</td>
</tr>
</tbody>
</table>

While the net pressure increased with modulus, it was relatively independent of the number of fractures present. This is rational, assuming that MShale uses the same net pressure calculation for different Young’s Modulus resulting in a linear relationship. Non-normalized data are provided in Table A1 in Appendix A.

4.2.3. Relationship between Young’s Modulus and Fracture Efficiency

The last Young’s Modulus relationship identified from this simulation involved the fracture efficiency of the system. For all simulations with the same number of major fractures, fracture efficiency decreased as a function of increasing Young’s Modulus. The reason for this is that increasing modulus leads to smaller width and larger surface area, increasing the surface over which fluid loss can occur. Presumably there are situations where combinations of modulus and fluid loss coefficient will change these relationships.
An example of this relationship can be seen in Table 5, using the data generated assuming that six major fractures were present.

Table 4: Example of Young’s Modulus / Fracture Efficiency Relationship

<table>
<thead>
<tr>
<th># of Fractures</th>
<th>Young’s Modulus of Shale (psi)</th>
<th>Fracture Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2.00E+06</td>
<td>36%</td>
</tr>
<tr>
<td>6</td>
<td>3.00E+06</td>
<td>34%</td>
</tr>
<tr>
<td>6</td>
<td>4.00E+06</td>
<td>32%</td>
</tr>
</tbody>
</table>

Similar trends can be observed for all simulations carried out where the number of major fractures was constant (refer to Table A1 in Appendix A).

4.3. Influence of the Number of Major Fractures on Hydraulic Fracturing Propagation

Three empirical relationships were established involving the number of major fractures present in the given system. These relationships are as follows:

1. The fracture height decreases as a function of the number of major fractures.
2. The net pressure of the system increases as a function of the number of major fractures.
3. The fracture efficiency increases as a function of the number of major fractures

4.3.1. Major Fractures and Fracture Height

It was determined that the height of the fracture(s) depended on the number of major fractures. By comparing values of the fracture height for larger numbers of major fractures taking fluid I presume to the values of the fracture height when fewer, parallel major fractures were taking fluid, it was clear the fracture height decreases with the number of major fractures present. Table 6 is an example.
Table 5: Example of Fracture Height Relationship with Number of Major Vertical Fractures

<table>
<thead>
<tr>
<th>Number of Major Vertical Fractures</th>
<th>Young’s Modulus (psi)</th>
<th>Fracture Height (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.00 x 10^6</td>
<td>720.8</td>
</tr>
<tr>
<td>3</td>
<td>3.00 x 10^6</td>
<td>690.6</td>
</tr>
<tr>
<td>6</td>
<td>3.00 x 10^6</td>
<td>665.5</td>
</tr>
<tr>
<td>9</td>
<td>3.00 x 10^6</td>
<td>648.3</td>
</tr>
<tr>
<td>12</td>
<td>3.00 x 10^6</td>
<td>635.0</td>
</tr>
<tr>
<td>15</td>
<td>3.00 x 10^6</td>
<td>624.0</td>
</tr>
</tbody>
</table>

Table 6 typifies the relationship between an increase in the number of major fractures and a decrease in the fracture height (Table A1 available in Appendix A). This is intuitive since, flow will be partitioned to more fractures and vertical and lateral extent will be smaller.

4.3.2. Major Fractures and Net Pressure

After observing the relationship described in 4.3.1 another relationship was identified between the number of major fractures and the net pressure. The height of each of the fracture(s) depended on the net pressure. The net pressure increases as a function of the number of major fractures present. This relationship can be seen by comparing net pressures with the number of major fractures present with the Young’s Modulus being held constant. Table 7 shows an example for a Young’s modulus of 3 x 10^6 psi. Refer to Appendix A for additional data.
Table 6: Example of Net Pressure relationship to the number of Major Vertical Fractures

<table>
<thead>
<tr>
<th>number of Major Vertical Fractures</th>
<th>Young’s Modulus of Shale (psi)</th>
<th>Net Pressure (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.00 x 10^6</td>
<td>66.3</td>
</tr>
<tr>
<td>3</td>
<td>3.00 x 10^6</td>
<td>111.4</td>
</tr>
<tr>
<td>6</td>
<td>3.00 x 10^6</td>
<td>154.9</td>
</tr>
<tr>
<td>9</td>
<td>3.00 x 10^6</td>
<td>188.9</td>
</tr>
<tr>
<td>12</td>
<td>3.00 x 10^6</td>
<td>217.9</td>
</tr>
<tr>
<td>15</td>
<td>3.00 x 10^6</td>
<td>243.9</td>
</tr>
</tbody>
</table>

4.3.3. Major Fractures and Fracture Efficiency

The number of major fractures correlates with the fracture efficiency of the system. For all simulations where the Young’s Modulus was held constant the fracture efficiency increased as a function of the number of major fractures that were present – more fractures, generally more surface area for fluid loss and consequently lower efficiency. An example of this relationship can be seen in Table 8.

Table 7: Major Fractures/Fracture Efficiency Relationship

<table>
<thead>
<tr>
<th>number of Major Vertical Fractures</th>
<th>Young’s Modulus of Shale (psi)</th>
<th>Fracture Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.00 x 10^6</td>
<td>19%</td>
</tr>
<tr>
<td>3</td>
<td>3.00 x 10^6</td>
<td>27%</td>
</tr>
<tr>
<td>6</td>
<td>3.00 x 10^6</td>
<td>34%</td>
</tr>
<tr>
<td>9</td>
<td>3.00 x 10^6</td>
<td>38%</td>
</tr>
<tr>
<td>12</td>
<td>3.00 x 10^6</td>
<td>41%</td>
</tr>
<tr>
<td>15</td>
<td>3.00 x 10^6</td>
<td>43%</td>
</tr>
</tbody>
</table>
Similar trends can be observed for all instances where Young’s Modulus was held constant. For more examples of this relationship refer to Table A1 in Appendix A.

4.4. Optimization of Hydraulic Fracturing through New Empirical Correlation

The most important discovery from this simulation work was a new empirical correlation that can be used to optimize hydraulic fracturing. At the heart of this new correlation is a dimensionless number that will be referred to as the Modified Young’s Modulus. The Modified Young’s Modulus is represented by the variable \( m_{YM} \). The Modified Young’s Modulus is defined as:

\[
m_{YM} = \frac{E}{p_{net}}\tag{1}
\]

By plotting the fracture height vs. \( m_{YM} \) nonlinear regression was used to solve a hydraulic fracturing optimization equation. An example of this can be seen in Figure 15.

Figure 15: Plot of Fracture Height vs. Modified Young’s Modulus (mYM)

Figure 15 was created by plotting all of the available fracture heights for a given number of major fractures for the three tested Young’s Modulus values. After all of the
data were plotted MS Excel was used to create a trendline through the data points. It was determined that the best fit for these data was a logarithmic function.

When determining which correlation would work best, trendlines were created for all available data points. Next the forecast graphical feature in MS Excel was used to extrapolate a wide range of values to generate a complete curve. By using statistical analysis it was determined that the data for one major fracture did not correlate with the other data points and therefore is represented by its own unique equation. The one major fracture trendline is represented in Figure 15 by an orange line. By using the data points where there was one major fracture present a trendline was added and an empirical correlation was solved for using MS Excel to be:

\[
Fracture Height = 68.86 \ln(mYM) - 17.467
\]  
(2)

or

\[
mYM = \exp \left( \frac{Fracture Height + 17.467}{64.84} \right)
\]  
(3)

The \( R^2 \) value for this non-linear regression was 0.9987. The fracture height defined in (2) and (3) is only valid for when there is one desired major fracture in the system.

For systems where multiple major fractures are present another empirical correlation was derived. By using statistical analysis, as well as graphically (Figure 15), most of the other scenarios (variation in the number of adjacent major fractures) produced curves that were very similar to each other. An example of this would be the trendlines for 9, 12, and 15 major fractures. Using this method it was determined that the most conservative empirical correlation for all multiple fracture scenarios was the logarithmic equation for the data points where there were 12 major fractures present. This equation from MS Excel is defined in Equations (4) and (5).
\[ Fracture\ Height = 105.31\ln(mYM) - 368.77 \] (4)

or

\[ mYM = \exp\left(\frac{Fracture\ Height + 368.77}{105.31}\right) \] (5)

The \( R^2 \) value for this non-linear regression was 0.9997. The Fracture Height defined in (4) and (5) is only valid for when there are multiple fractures in the system and no stress restrictions on fracture growth, for the viscosity and rate selected in the analyses and similar fluid loss.

After analyzing the data provided above in Excel the same data set was analyzed in Matlab and a regression equation was solved using Matlab’s nonlinear fit function. Matlab was used to get a more robust equation as well as to solve for 95% confidence intervals. By using Matlab’s nonlinear fit function on the data points where there were 12 major fractures present Equations (6) and (7) were defined:

\[ Fracture\ Height(95\%\ CI) = 108.5 \pm 1.4\ln(mYM) - 399.1 \pm 13.4 \] (6)

or

\[ mYM = \exp\left(\frac{Fracture\ Height + 399.1 \pm 13.4}{108.5 \pm 1.4}\right) \] (7)

The \( R^2 \) value for this non-linear regression was 0.9999. The fracture height defined in (6) and (7) is only valid for when there are multiple fractures in the system. Due to the addition of confidence intervals it is recommended that (6) or (7) be used over (4) or (5).

Assuming radial growth with the fluid loss, treating fluid and rate considered here, a stimulation engineer can infer fracture height and length from the measured net pressure and an estimate or measurement of Young’s modulus.
(preferably the plane strain Young’s modulus). Once the target net pressure is calculated the number of vertical fractures for this hydraulic fracturing job can also be estimated by using Table B1 in Appendix B. By repeating this for other fluids, rates and stress conditions, overall empirical relationships can be developed (see Hai Zu Meng, circa 1985).

5. Conclusions and Future Recommendations

This research was successful in optimizing hydraulic fracturing by using MShale. Six relationships were determined by analyzing data for various numbers of major fracture(s), Young’s Modulus, fracture height, and the net pressure of the hydraulic fracturing process. These trends are:

1. The fracture height increases with Young’s Modulus.
2. The net pressure of the system increases with Young’s Modulus.
3. The fracture efficiency decreases with Young’s Modulus.
4. The fracture height decreases with an increasing number of major fractures.
5. The net pressure of the system increases with the number of major fractures.
6. The fracture efficiency increases as a function of the number of major fractures.

An empirical correlation was derived from the different simulation runs and quantified using a new dimensionless number, the Modified Young’s Modulus (mYM). By plotting the fracture height vs. mYM, a logarithmic equation was regressed using MS Excel’s trendline feature. After some algebraic manipulation the mYM can be solved for any desired value of fracture height using Equation (3). From the mYM number and a known/inferred Young’s Modulus the net pressure can be estimated. After estimating a value for the net pressure the number of major fractures can then be estimated. Correlations such as these could optimize hydraulic fracturing
calculations tremendously and will help operators make decisions on the fly. This new empirical correlation represented by (5) and (6) can be used to solve for the desired fracture heights in the Bakken, Barnett, and Marcellus shale formations since it has been extrapolated to cover all Young’s Modulus ranges defined in 1.3.

5.1. Future Recommendations

Although positive results were obtained the main objective of optimizing the spacing of hydraulic fracturing systems in horizontal wells was not accomplished since it was not possible to predict fracture interactions between different clusters. It would be feasible to model fracture interactions using equations from Yew and Li’s 1988 paper titled “Fracturing of a Deviated Well,” SPE number 16930 and their 1989 paper titled “On Fracture Design of Deviated Wells,” SPE number 19722.

The last recommendation for future work would be to calculate the fracture length, height, and net pressure values by using the Young’s Modulus of the shale formations mentioned in chapter 1 and then compare those values to the extrapolated answers. This would develop a Young’s Modulus range where the correlation is valid. In addition to this it is recommended that this simulation work be redone using a 3D fracture model and then solving for a similar correlation as to the one provided in this paper would be invaluable in understanding how fractures in industry propagate and interact.
### Table of Nomenclature

<table>
<thead>
<tr>
<th>Latin Symbols</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>Fracture Height</td>
<td>ft</td>
</tr>
<tr>
<td>k</td>
<td>Permeability</td>
<td>mD</td>
</tr>
<tr>
<td>(mYM)</td>
<td>Young’s Modulus / Net Pressure</td>
<td>dimensionless</td>
</tr>
<tr>
<td>(P_{\text{net}})</td>
<td>Net Pressure</td>
<td>psi</td>
</tr>
<tr>
<td>(x_f)</td>
<td>Fracture Length</td>
<td>ft</td>
</tr>
<tr>
<td>(Y_{YM})</td>
<td>Young’s Modulus (Modulus of Elasticity)</td>
<td>psi</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Greek Symbols</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta)</td>
<td>Fracture Spacing</td>
<td>ft</td>
</tr>
<tr>
<td>(\Delta h)</td>
<td>Change in height</td>
<td>ft</td>
</tr>
<tr>
<td>(\Delta P)</td>
<td>Change in pressure</td>
<td>psi</td>
</tr>
<tr>
<td>(\sigma_{\text{min}})</td>
<td>Minimum Horizontal Stress</td>
<td>psi</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Porosity</td>
<td>%</td>
</tr>
</tbody>
</table>
# Appendix A

## Raw Data

### Table A1: Raw data for the MShale Hydraulic Fracturing Optimization Simulations

<table>
<thead>
<tr>
<th>Run #</th>
<th># of Major Vertical Fractures</th>
<th>Young’s Modulus of Shale (psi)</th>
<th>Net Pressure (psi)</th>
<th>Fracture Length (ft)</th>
<th>Fracture Height (ft)</th>
<th>Fracture Efficiency</th>
<th>Young’s Modulus / Net Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2.00E+06</td>
<td>49.7</td>
<td>355.3</td>
<td>712.3</td>
<td>22%</td>
<td>4.02E+04</td>
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<tr>
<td>2</td>
<td>1</td>
<td>3.00E+06</td>
<td>66.3</td>
<td>359.8</td>
<td>720.8</td>
<td>19%</td>
<td>4.53E+04</td>
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<tr>
<td>3</td>
<td>1</td>
<td>4.00E+06</td>
<td>81.2</td>
<td>362.5</td>
<td>726.2</td>
<td>18%</td>
<td>4.93E+04</td>
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<tr>
<td>4</td>
<td>3</td>
<td>2.00E+06</td>
<td>82.9</td>
<td>338.8</td>
<td>680.8</td>
<td>30%</td>
<td>2.41E+04</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>3.00E+06</td>
<td>111.4</td>
<td>343.9</td>
<td>690.6</td>
<td>27%</td>
<td>2.69E+04</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>4.00E+06</td>
<td>137.3</td>
<td>347.2</td>
<td>696.8</td>
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<td>2.91E+04</td>
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<tr>
<td>7</td>
<td>6</td>
<td>2.00E+06</td>
<td>115.6</td>
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<td>1.73E+04</td>
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<td>330.7</td>
<td>665.5</td>
<td>34%</td>
<td>1.94E+04</td>
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<td>4.00E+06</td>
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<td>334.5</td>
<td>672.8</td>
<td>32%</td>
<td>2.10E+04</td>
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<td>1.59E+04</td>
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<td>1.72E+04</td>
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<td>307.5</td>
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<td>1.23E+04</td>
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<tr>
<td>14</td>
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<td>314.3</td>
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<td>1.38E+04</td>
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<td>1.49E+04</td>
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<td>301.3</td>
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<td>46%</td>
<td>1.09E+04</td>
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<td>17</td>
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<td>243.9</td>
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<td>15</td>
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<td>299.4</td>
<td>313.0</td>
<td>632.7</td>
<td>41%</td>
<td>1.34E+04</td>
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</tbody>
</table>
Appendix B

Empirical Correlation Table for Multiple Fractures

Table B1: Empirical Correlation Table to solve for the Number of Major Vertical Fractures using (5) or (6)

<table>
<thead>
<tr>
<th># of Major Vertical Fractures</th>
<th>Young’s Modulus of Shale (psi)</th>
<th>Net Pressure (psi)</th>
<th>Fracture Length (ft)</th>
<th>Fracture Height (ft)</th>
<th>Fracture Efficiency</th>
<th>Young’s Modulus / Net Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.00E+06</td>
<td>82.9</td>
<td>338.8</td>
<td>680.8</td>
<td>30%</td>
<td>2.41E+04</td>
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<tr>
<td>6</td>
<td>2.00E+06</td>
<td>115.6</td>
<td>324.7</td>
<td>654.4</td>
<td>36%</td>
<td>1.73E+04</td>
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<td>9</td>
<td>2.00E+06</td>
<td>141.2</td>
<td>315.0</td>
<td>636.4</td>
<td>40%</td>
<td>1.42E+04</td>
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<td>2.00E+06</td>
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<td>307.5</td>
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<td>6</td>
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<td>330.7</td>
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<td>1.94E+04</td>
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<td>321.5</td>
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<td>38%</td>
<td>1.59E+04</td>
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<tr>
<td>12</td>
<td>4.00E+06</td>
<td>267.7</td>
<td>318.8</td>
<td>643.3</td>
<td>39%</td>
<td>1.49E+04</td>
</tr>
<tr>
<td>15</td>
<td>4.00E+06</td>
<td>299.4</td>
<td>313.0</td>
<td>632.7</td>
<td>41%</td>
<td>1.34E+04</td>
</tr>
</tbody>
</table>
Appendix C

Confidence Interval

The 95% Confidence Interval, CI, was solved for using Matlab. The code that produced equation (5) and (6) is as follows:

% This code was written by Mike Hansen
% University of Utah Dept. of Chemical Engineering
% PhD Candidate
% 4/2/2013

clc;
clear all;
% Define the fracture heights
y = [622.5; 635.0; 643.3];
% Define the Modified Young’s Modulus
h = [1.23E+04; 1.38E+04; 1.49E+04];
% Set up anonymous logarithmic function based on MS Excel Data
f = @(c,h)c(1)*log(h) + c(2);
[c, r, j, covb, mse] = nlinfit(h,y,f,[100,-300]);
% Solve for Confidence Intervals
ci = nlinfit(c,r,'covar',covb);
hc = linspace(100,90000,50)';
plot(h,y,'o',hc,f(c,hc));
Appendix D

Sample Net Pressure Calculation using Equation (7)

Problem: Solve for the Net Pressure required to create a fracture that is 700 ft high

Given: The Young’s Modulus of the shale formation is 2e+06 psi

Use (6) which is defined as:

\[ mYM = EXP(\frac{Fracture Height + 399.1 \pm 13.4}{108.5 \pm 1.4}) \ (95\% \ CI) \] (7)

Solve for Modified Young’s Modulus, mYM:

\[ mYM = EXP\left(\frac{700 + 399.1 \pm 13.4}{108.5 \pm 1.4}\right) = 25,063 \pm 180.35 \ (95\% \ CI) \] (D-1)

Recall that:

\[ mYM = \frac{YYM}{P_{net}} \] (D-2)

Solve for \( P_{net} \)

\[ P_{net} = \frac{YYM}{mYM} \] (D-3)

\[ P_{net} = \frac{2e+06 \ psi}{25,063 \pm 180.35} = 79.8 \pm 0.57 \ psig \ (95\% \ CI) \] (D-4)

Once the target net pressure is calculated the number of vertical fractures for this hydraulic fracturing job can be calculated using Table B1 in Appendix B. By doing this the number of major fractures should be approximately 3.
Appendix E

mShale™ Inputs for Simulation

Data Options

The simulation method was set to *Design Mode* with the *Reservoir Coupling* option set to *Linear*. The *Fluid Loss Model* was set to *Constant* and the fracture treatment design was set to input. The *Wellbore Hydraulics Method* was set to *Empirical* and the *total number of iterations* for the Fracture Solution was set to 100 with a maximum time step of 1. Heat transfer of the system was turned off. A screenshot of the general tab screen in Data Options is shown in Figure 3:

![Data Options Window, General Tab Settings](image)

Figure E1: Data Options Window, General Tab Settings

After the general simulation criteria have been set, the next step is to click on the fracture tab within the Data Options window to set the fracture properties. For this simulation the *Fracture Geometry* was set to *Vertical Ellipsoidal*, the *Flowback* was set to off and the Simulate to Closure
was set to off. The Propagation Parameters were set to Default Growth (+,-) and the Fracture Initiation Interval was set to Perforated Interval. For simplicity, the Fracture Friction Model was turned off as well as the Wall Roughness and Tip Effect options. A screenshot is provided in Figure 4:

![Data Options Window, Fracture Tab Settings](image)

**Figure E2: Data Options Window, Fracture Tab Settings**

The last necessary setup input for the Data Options window is the proppant criteria. For this simulation, the Proppant Solution, Proppant Ramp, and Proppant Flowback were all set to off. The Perforation Erosion, Wellbore-Proppant Effects, and the Fracture Proppant Effects were set to None. During the simulated hydraulic fracturing treatment it is assumed that the shale formation will be fractured with 2% KCl in water and there will be no proppant in the system. In reality, proppant will be pumped. A screenshot of the proppant parameters is provided in Figure 5:
Wellbore Hydraulics

After all of the parameters in the Data Options were selected, the next requirement was to define the Wellbore Hydraulics. The Wellbore Hydraulics window can be selected by clicking on Data and then selecting Wellbore Hydraulics. In the General tab, the Injection Down criterion was set to Casing and the Horizontal Well option was selected since the fractures will occur in the horizontal part of the wellbore. A screenshot of the General tab in the Wellbore Hydraulics window can be seen in Figure 6:
After defining the General Injection system, the deviation\(^1\) of the wellbore was defined using the Deviation tab. The Wellbore Survey Method was set to Tangential and the Deviation Input was selected to be Inclination Only (2D). Next, the Depths Entered Elsewhere option to keep the TVD Values was selected. Last the deviated wellbore parameters were selected by selecting a Measured Depth, \(ft\), the Inclination Angle, degrees, and allowing mShale to calculate the Total

\(^1\) Angle from the vertical
Vertical Depth, ft. The wellbore deviation that was used for this evaluation is tabulated in Table 1.

Table E1: Theoretical Wellbore Deviation Parameters

<table>
<thead>
<tr>
<th></th>
<th>Measured Depth (ft)</th>
<th>Inclination Angle (degrees)</th>
<th>TVD (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5000</td>
<td>0</td>
<td>5000</td>
</tr>
<tr>
<td>2</td>
<td>6500</td>
<td>17.2674</td>
<td>6432.39</td>
</tr>
<tr>
<td>3</td>
<td>7250</td>
<td>45.6486</td>
<td>6956.69</td>
</tr>
<tr>
<td>4</td>
<td>7419.27</td>
<td>75.1747</td>
<td>7000</td>
</tr>
<tr>
<td>5</td>
<td>9000</td>
<td>90</td>
<td>7000</td>
</tr>
</tbody>
</table>

After setting the wellbores deviation trajectory, the General wellbore geometry and casing program were defined. The generic wellbore was constructed as an horizontal lateral at a Total Vertical Depth, TVD, of 7,000 ft with the toe (remote end) of the well having a Measured Depth, MD of 9,000 ft (2,000 ft of horizontal extent in the shale). The wellbore was then cased using with 7-inch. OD steel pipe with a weight of 33.7 \( \frac{lb}{ft} \) and a 6.048 in. ID. The stimulation would be pumped inside this casing and no tubing would be used in these simulations. A plot of the theoretical wellbore can be observed below in Figure 7.
With the wellbore and wellbore hydraulics defined, the next step was to specify the zones where the hydraulic fractures will occur. This can be done by clicking on Data and then selecting the Zones feature. The Zones window will open and request that the user name the zone and define the Center of Perfs MD (ft), Top of Fracture Initiation TVD (ft), Bottom of Fracture Initiation TVD (ft), and Ellipsoidal Aspect Ratio. The Center of Perfs TVD (ft) column is not selectable since
mShale will calculate this value from the provided *Center of Perfs MD (ft)* information. For this simulation the parameters defined are shown in Figure 8.

![Figure E6](image)

**Figure E6: Zones Window with all simulation inputs**

For this simulation only one fracture zone was needed, if multiple fractures zones are to be simulated make sure that the corresponding box in the active column is checked.

After all the necessary information has been input into the *Zones* window, the specific *Zone Data* are entered. The *Zone Data* window can be opened by clicking on the box labeled … in the Zone Data column. When the *Zone Data* window opens MShale will automatically open the *Perforations* tab window. For this simulation, the number of perforations was set to 5 and the diameter of each of these perforations was preset to 0.25 inches (refer to Figure 9).
After putting all of the required inputs in the *Perforations* window, the *Pay Zone* is selected. In this window the permeability of the formation where the hydraulic fracture will occur and the general location of the hydraulic fracturing fluid pay zone can be defined. The permeability has been set to 0.001 mD. When entering the hydraulic fracturing pay zone information for a horizontal well it was noted that MShale was incapable of selecting the pay zone where the TVD of the *Depth From* was equal to the TVD of the *Depth To* even though technically they would be the same in a true horizontal wellbore. In order to overcome this limitation in MShale the *From TVD* value was set to 8850 ft and the *To TVD* was set to 6900 ft. When specifying the TVD values the MD values are calculated by MShale. The input screen for the Pay Zone tab for this simulation is represented in Figure 10.
Once the Pay Zone tab was completed, the next input used the Fracture Network Options tab. The Multiple Fractures option was selected. Next the Characteristics tab was opened and the Number of Major Vertical Fractures and Spacing along the Minor Axis were defined. The Number of Major Vertical Fractures for this simulation varied from 1 to 15 in order to study fracture interactions. After defining all parameters in the Characteristics tab, the Interaction tab was opened. The Fracture Interaction was set to “Full.” By selecting “Full” MShale assumes that there will be stiffness or fluid loss interactions between fractures. Next, the Proppant Distribution tab was selected and Uniform Distribution Style was chosen for this simulation. Finally, the Near Wellbore tab was opened and Ignore Table was selected, indicating to MShale to ignore near well effects. Since this simulation is for a theoretical wellbore, no specific near wellbore effects were available to serve as an input.
Treatment Schedule

After all of the Zone data had been entered the desired Treatment Schedule was specified. This window can be opened by clicking on the Data tab on the top of MShale and then selecting Treatment Schedule. When the Treatment Schedule window opens it will default to the General tab. Schedule Type was set to Surface and Stage Friction Multipliers was selected. The Wellbore Volume is calculated by MShale using the defined wellbore characteristics in the Wellbore Hydraulics window. The Fraction of the Well Filled and the Wellbore Fluid Friction Multiplier were both set to 1 for this simulation. For more information about the General tab of the Treatment Schedule window please refer to Figure 11.

![Figure E9: General tab of the Treatment Schedule window](image)

Once the General tab of the Treatment Schedule window has been completed the States tab is opened. The Wellbore Fluid Type was set to D045, WF250, 2% KCl w/ 0.25 gal/1000 J134L breaker, from the MShale Fluid database. The prescribed treatment schedule, indicating which fluids were pumped, at what rates and volumes of each fracturing stage, is shown in Figure 12.
Figure E10: Stages tab of the Treatment Schedule window

**Rock Properties**

The next step is defining the *Rock Properties* for the theoretical geologic formations being considered. To open this window, click on the *Data* tab and select *Rock Properties*. The formations above and below the shale formation in were considered to be limestone. Fracture growth vertical out of the shale was prevented because a two-dimensional simulation had been selected.

Young’s Modulus of the shale was varied from $2 \times 10^6$ to $4 \times 10^6$ (psi) in $1e+06$ increments as part of this simulation. Figure 15 is a screenshot of typical *Rock Properties* input.

Figure E11: Simulation inputs for the Rock Properties window

**Fluid Loss**

The last input needed for this simulation is found in the Fluid Loss window. This window can be opened by clicking on the Data tab and then selecting the Fluid Loss window. For this
simulation the Leakoff Coefficient was set to 0.001 \( \left( \frac{ft}{min^{1.2}} \right) \) and the Spurt Loss was assumed to be zero. For more details about the setup of the Fluid Loss window please refer to Figure 14 below:

![Fluid Loss window](image)

**Figure E12:** Simulation inputs for the Fluid Loss window
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